PROBLEM SET 4 SOLUTIONS
Physics 2021

1. $P^2 = a^3$ $\quad a = \text{cube-root}\{ (25)^2 \} = \text{cube-root}\{ 625 \} = 8.55$ AU

2a. Major axis is the sum of the perihelion and aphelion distances

Semi-major axis = a = (2 + 6) / 2 = 4 AU

2b. $(\text{Period})^2 = (\text{Semi-major axis})^3$

Period = P = SQRT(a^3) = SQRT(4^3) = 8 yr

3a. $P^2 = a^3$ $\quad a^3 = (64)^2$ $\quad a = (64)^{2/3} = 16$ AU

3b. Perihelion distance = major axis – aphelion distance

= (2 x 16 AU) – 31.5 AU = 0.5 AU

4. $W = [G m (4 \mathcal{M}) / (4R)^2] \; / \; [G m (\mathcal{M}) / (R)^2] = 3 / (4)^2 = 1/4$

You would weigh one-fourth of your Earth weight.

5. Mass is 100X greater but distance^2 = 10 x 10 = 100X weaker, so net effect = 1

6a. Period = 1 solar day = 24 hours

6b. To get the distance use Kepler's Third Law, but convert days to years, kg to solar masses, and km to AU. Then convert the answer back to km.

$P = 1 \text{ d} / 365.25 \text{ d/yr} = 2.738 \times 10^{-3}$ yr

$\mathcal{M} = 5.974 \times 10^{24}$ kg / 1.989×10^{30} kg = 3.00×10^{-6} solar masses

$a^3 = \mathcal{M} P^2 = (3.00 \times 10^{-6} \text{ solar masses}) (2.738 \times 10^{-3} \text{ yr})^2$

$= 2.249 \times 10^{-11}$ AU ^{3}

$a = 2.822 \times 10^{-4}$ AU = 42,300 km

7a. $S = 1.25$ yr

$$P^{-1} = E^{-1} - S^{-1} = 1 - (1.25)^{-1} = 0.2$$

$$P = 1 / 0.2 = 5 \text{ years}$$

7b. $P^2 = a^3$ $\qquad a = P^{2/3} = (5)^{2/3} = 2.92$ AU

8. $d_{aph}\, v_{aph} = d_{peri}\, v_{peri}$

$$(d_{aph} / d_{peri}) = v_{peri} / v_{aph}$$

$3 = 25$ km/s $/ v_{aph}$ $\qquad v_{aph} = 25$ km/s $/ 3 = 8.3$ km/s

9. $(\mathcal{M}_1 + \mathcal{M}_2)\, P^2 = a^3$

$$(2\mathcal{M}_1 + 2\mathcal{M}_2)\, P_{new}^2 = 2(\mathcal{M}_1 + \mathcal{M}_2)\, P_{new}^2 = (2a)^3 = 8\, a^3$$

$(\mathcal{M}_1 + \mathcal{M}_2)\, P_{new}^2 = 4\, a^3$ $\qquad P_{new} = 2\, P$

10. $\mathcal{M}_1 = a^3 / P^2$

$P^2 = a^3 / \mathcal{M}_1 = (1 \text{ AU})^3 / 2 = 0.5$ $\qquad P = 0.71$ yr