

PROBLEM SET 8 SOLUTIONS

Physics 2021

1a. Core

$$R_{\text{core}} = 3500 \text{ km} \quad R_{\text{earth}} = 6350 \text{ km}$$

$$[R_{\text{core}}^3 / R_{\text{earth}}^3] = [3500^3 / 6350^3] = 0.17 = 17\%$$

1b. Mantle

$$R_{\text{core}} = 3500 \text{ km} \quad R_{\text{mantle}} = 6320 \text{ km} \quad R_{\text{earth}} = 6350 \text{ km}$$

$$[(R_{\text{mantle}}^3 - R_{\text{core}}^3) / R_{\text{earth}}^3] = [(6320^3 - 3500^3) / 6350^3] = 0.82 = 82\%$$

1c. Crust

$$1.00 - (0.17 + 0.82) = 0.01 = 1\%$$

2. $V = 4\pi/3 R^3 = (4\pi/3) (1300 \times 10^3 \text{ m})^3 = 9.2 \times 10^{18} \text{ m}^3$

$$\mathcal{M} = \rho V = (13,000 \text{ kg/m}^3) (9.2 \times 10^{18} \text{ m}^3)$$

$$\mathcal{M} = 1.2 \times 10^{23} \text{ kg} \quad \mathcal{M}_{\text{earth}} = 5.94 \times 10^{24} \text{ kg}$$

$$\mathcal{M} / \mathcal{M}_{\text{earth}} = 1.2 \times 10^{23} \text{ kg} / 5.97 \times 10^{24} \text{ kg} = 0.02$$

3. $\mathcal{M}_{\text{earth}} = 5.97 \times 10^{24} \text{ kg} \quad R_{\text{earth}} = 12,756 \text{ km} / 2 = 6.378 \times 10^6 \text{ m}$

$$\rho = \mathcal{M} / V = (5.97 \times 10^{24} \text{ kg}) / [(4\pi/3) (6.378 \times 10^6 \text{ m})^3]$$

$$= 5,500 \text{ kg/m}^3 = 5.5 \text{ g/cm}^3$$

4. The distance from South America to Africa is approximately 6,600 km.

$$d = v t \quad t = (6,600 \text{ km} \times 10^5 \text{ cm/km}) / (3 \text{ cm/yr}) = 220 \text{ million years}$$

5. Hydrogen either escaped or combined with oxygen to form water.
Carbon combined with oxygen to form carbon dioxide; most is trapped in rocks.
Nitrogen combined with hydrogen to form ammonia or combined with itself (N₂).

6a. Albedo = 0.39

$$\text{Light reflected} = (0.39) (1.75 \times 10^{17} \text{ W}) = 6.8 \times 10^{16} \text{ W}$$

6b. Light absorbed = (0.61) (1.75 x 10¹⁷ W) = 1.07 x 10¹⁷ W

6c. R_{earth} = 6.38 x 10⁶ m

$$\text{Surface Area} = 4 \pi r^2 = 4 \pi (6.38 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$$

$$\text{Power radiated per m}^2 = (1.07 \times 10^{17} \text{ W}) / (5.1 \times 10^{14} \text{ m}^2)$$

$$= 209 \text{ W / m}^2$$

6d. Stefan-Boltzmann Law $F = \sigma T^4$

$$T = [F / \sigma]^{0.25} = [209 \text{ W/m}^2 / 5.67 \times 10^{-8} \text{ W /m}^2 \text{ K}]^{0.25}$$

$$= 246 \text{ K} = -27^\circ\text{C}$$

- 6e. The Earth's temperature is higher due to the greenhouse effect.