## PROBLEM SET 13 SOLUTIONS

## Physics 2021

1. 

|  | Rhea |
| :--- | :---: |
| $\mathbf{a ~ ( k m ) ~}$ | 527,108 |
| $\mathbf{a}(\mathbf{A U})$ | $3.52 \times 10^{-3}$ |
| $\mathbf{a}^{3}$ | $4.34 \times 10^{-8}$ |
| $\mathbf{P}$ (days) | 4.518 |
| $\mathbf{P}(\mathbf{y r})$ | $1.24 \times 10^{-2}$ |
| $\mathbf{P}^{2}$ | $1.53 \times 10^{-4}$ |
| $\mathscr{M}^{2} \mathbf{P}^{2}$ | $4.35 \times 10^{-8}$ |

$\mathbf{a}^{3}=\mathcal{M} \mathbf{P}^{2} \quad$ where $\mathcal{M}=$ Mass of Saturn / Mass of the Sun $=568.3 \times 10^{24} \mathrm{~kg} / 2 \times 10^{30} \mathrm{~kg}=2.84 \times 10^{-4}$.
2. $\quad P_{1}=3 / 5 P_{2} \quad$ From the perspective of \#1, \#2 is a superior planet.
$\mathbf{P}_{\mathbf{2}}=1.67 \mathrm{P}_{1}$
Superior Planet: $\mathbf{1 / P}=\mathbf{1} / \mathrm{E}-1 / \mathrm{S}$
$1 / \mathrm{S}=1 / 1-1 / 1.67=1.00-0.60=0.40 \mathrm{yr} \quad \mathrm{S}=2.5$ orbits
3. Jupiter's rotation is faster than Io's revolution, so the magnetic field overtakes Io (catching up from the back first).
4. Total mass $=(0.15)\left(8.94 \times 10^{22}\right)=1.34 \times 10^{22} \mathrm{~kg}$

$$
\begin{aligned}
\text { Time }= & \left(1.34 \times 10^{22} \mathrm{~kg}\right) /(1000 \mathrm{~kg} / \mathrm{s})=1.34 \times 10^{19} \text { seconds } \\
= & 4.25 \times 10^{11} \text { years }=425 \text { billion years (much greater than the age of the } \\
& \text { Solar System })
\end{aligned}
$$

5. More of the atmosphere would be lost because the Earth-Moon system is in a warmer region of the Solar System. Because Titan is larger in diameter than the Moon, solar eclipses would be more common.

6a. $\quad P_{n}=163.73 \mathrm{yr}$ $P_{p}=248.0 \mathrm{yr}$

$$
P_{n} / P_{p}=163.73 / 248.0=0.66=2: 3
$$

6b. The shapes of their orbits can be from circular to very elliptical, and still have similar periods and semi-major axes. Also, their orbits can be in different planes. Third, their sizes and masses are rather small. All of these contribute to making it very difficult for two of these worlds to interact, much less collide.

7a. $\quad d_{p}=29.70 \mathrm{AU}$
Brightness $=(1 / 29.70)^{2}=0.00113$
7b. $\quad d_{a}=49.39 \mathrm{AU}$
Brightness $=(1 / 49.39)^{2}=0.00041$
7c. $\quad$ Ratio $=0.00113 / 0.00041=2.8 \mathrm{X}$
8. $\quad D=19,640 \mathrm{~km}$

$$
\begin{aligned}
& \mathrm{d}=4.435 \times 10^{9} \mathrm{~km}-1.5 \times 10^{8} \mathrm{~km}=4.285 \times 10^{9} \mathrm{~km} \\
& \alpha=206,265(\mathrm{D} / \mathrm{d})=(206,265)(19,640 \mathrm{~km}) /\left(4.285 \times 10^{9} \mathrm{~km}\right) \\
& \quad \alpha=0.95 \mathrm{arcsec}
\end{aligned}
$$

9. $\lambda_{\text {max }}=0.0029 / T=(0.0029) /(55 \mathrm{~K})=5.3 \times 10^{-5} \mathrm{~m}=53 \mu \mathrm{~m}$

Problems 4, 6a, and 9 are worth one point each;
Problems 7(ab) and 8 are worth two points each;
Problem 1 is worth three points.

