

SOLUTION SET 3

Physics 2022

1. $F = 2f$ $M = F/f = 2f/f = 2$

2. $\text{Fraction} = \text{Area (cage)} / \text{Area (mirror)} = \pi R^2(\text{cage}) / \pi R^2(\text{mirror})$
 $= (1/5)^2 = 1/25 = 0.04$

3. $\text{LGA} \propto \text{Area}$

$$\text{LGA}(1) \propto (300 \text{ m})^2 \qquad \text{LGA}(2) \propto (50 \text{ m})^2$$

$$\text{LGA}(1) / \text{LGA}(2) = (300 / 50)^2 = (6)^2 = 36 \text{ times}$$

4. $\text{LGA}(1) / \text{LGA}(2) = (\text{Diameter} / 1)^2 = \text{Diameter}^2 = 1000 \text{ m}^2$
 $\text{Diameter} = 31.6 \text{ m}$

5a. $\text{Light Gathering Power} = \text{Area (Subaru)} / \text{Area (Hubble)}$

$$= \pi R^2(\text{Subaru}) / \pi R^2(\text{Hubble}) = (8.3 \text{ m} / 2.4 \text{ m})^2 = 12.0 \text{ X}$$

5b. Subaru has a larger aperture, so it can detect fainter objects.

Hubble is outside the Earth's atmosphere, so it achieves a higher resolution.

6a. $\text{Magnification} = F/f = 2 \text{ m} / 9\text{mm} = 2000 \text{ mm} / 9\text{mm} = 222 \text{ X}$

6b. $\text{Magnification} = F/f = 2000 \text{ mm} / 20 \text{ mm} = 100 \text{ X}$

6c. $\text{Magnification} = F/f = 2000 \text{ mm} / 55 \text{ mm} = 36.4 \text{ X}$

6d. $\alpha = 2.5 \times 10^5 \lambda / D = 2.5 \times 10^5 / 0.20 \lambda = 1.25 \times 10^6 \lambda$

where λ and D are in meters

$$\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m then } \alpha = (1.25 \times 10^6)(600 \times 10^{-9}) = 0.75 \text{ arcsec}$$

7a. $D = d \tan \alpha = (6.28 \times 10^8 \text{ km}) \tan [(0.1 \text{ arcsec}) / (60 \times 60)] = 304.5 \text{ km}$

Hubble's resolution on Io.

7b. $D = d \tan \alpha = (3.844 \times 10^5 \text{ km}) \tan [(1.0 \text{ arcmin}) / (60)] = 111.8 \text{ km}$

Human eye's resolution on the Moon.

8a. $D = d \tan \alpha = (7 \times 10^7 \text{ ly}) \tan (0.1 / 3600) = 34 \text{ ly}$

8b. $d = D / \tan \alpha = (1.8 \times 10^5 \text{ km}) / \tan (0.1 / 3600) = 37 \text{ km}$

9. $\alpha = 250,000 \lambda / D$ $\lambda_1 = 5 \times 10^{-7} \text{ m}$ $D_1 = 4 \text{ m}$ $\lambda_2 = 10^{-4} \text{ m}$

$$\lambda_1 / D_1 = \lambda_2 / D_2$$

$$D_2 = D_1 \lambda_2 / \lambda_1 = (4 \text{ m}) (10^{-4} \text{ m}) / (5 \times 10^{-7} \text{ m}) = 800 \text{ m}$$

10. $\alpha = 250,000 \lambda / D$

$$10^{-3} \text{ arcsec} = 250,000 (550 \times 10^{-9} \text{ m}) / D$$

$$D = 250,000 (550 \times 10^{-9} \text{ m}) / (10^{-3} \text{ arcsec})$$

$$D = 137.5 \text{ m}$$