SOLUTION SET 6
Physics 2022

1. $L_* = 1000\ L_{sun}$ $T_* = 1000$ K $T_{sun} = 5800$ K

$L_* / L_{sun} = (R_* / R_{sun})^2 (T_* / T_{sun})^4$

$1000 = (R_* / R_{sun})^2 (1000 / 5800)^4$

$R_* = 1064\ R_{sun} = 7.4 \times 10^8$ km $= 4.9$ AU

2. $L_{sun} = 3.85 \times 10^{26}$ J/s 1 reaction of H to He releases 26.7 MeV

Number of reactions $= (3.85 \times 10^{26}\ J/s) / (26.7 \times 10^6 \times 1.6 \times 10^{-19}\ J/eV)$

$= 9.0 \times 10^{37}$ reactions per second

Mass = (4 hydrogen atoms) (number of reactions)

$= 4\ (1.67 \times 10^{-27}\ kg)(9.0 \times 10^{37}$ reactions/second$) = 6.0 \times 10^{11}$ kg/s

3a. In one reaction of H burning, 4.031280 u release 26.71 MeV

$[(4.03128\ u)(1.66054 \times 10^{-27}\ kg/u)] / [(26.7 \times 10^6\ eV)(1.6 \times 10^{-19}\ J/eV)]$

$= 1.57 \times 10^{-15}$ kg/J

In one second, the Luminosity (energy) is 3.83×10^{26} J/s, so

H burned in one second $= (1.57 \times 10^{-15}\ kg/J)(3.83 \times 10^{26}\ J/s) = 6.0 \times 10^{11}$ kg/s

The main sequence lifetime of the Sun is 1.2×10^{10} years, so the total H burned is

Total H burned $= (6.0 \times 10^{11}\ kg/s)(1.2 \times 10^{10})(365.25)(24)(3600\ s)$

$= 2.27 \times 10^{29}$ kg

3b. $\mathcal{M}_{sun} = 1.99 \times 10^{30}$ kg The composition of the Sun is 74% Hydrogen.

$\mathcal{M}_H = (0.74)(1.99 \times 10^{30}\ kg) = 1.47 \times 10^{30}$ kg

Ratio $= (2.27 \times 10^{29}\ kg) / (1.47 \times 10^{30}\ kg) = 0.154$

4a. $E = \Delta m\, c^2 = (2 \times 10^{-26}\ kg)(3 \times 10^8\ m/s)^2 = 1.8 \times 10^{-9}\ J$

4b. $E = \Delta m\, c^2 = (1\ kg)(3 \times 10^8\ m/s)^2 = 9.0 \times 10^{16}\ J$

4c. $E = \Delta m\, c^2 = (6 \times 10^{+24}\ kg)(3 \times 10^8\ m/s)^2 = 5.4 \times 10^{41}\ J$

5a. $\tau = E / L_{sun} = (1.8 \times 10^{-9}\ J) / (3.9 \times 10^{26}\ J/s) = 4.6 \times 10^{-36}\ s$

5b. $\tau = E / L_{sun} = (9.0 \times 10^{16}\ J) / (3.9 \times 10^{26}\ J/s) = 2.3 \times 10^{-10}\ s$

5c. $\tau = E / L_{sun} = (5.4 \times 10^{41}\ J) / (3.9 \times 10^{26}\ J/s) = 1.38 \times 10^{15}\ s = 44$ million years

6a. $E = \Delta mc^2 = 2 \times (9.1 \times 10^{-31}\ kg) \times (3 \times 10^8\ m/s)^2 = 1.6 \times 10^{-13}\ J$

6b. $E = hc / \lambda$ Each photon gets half of the energy released $= 0.8 \times 10^{-13}\ J$

$\lambda = (6.6 \times 10^{-34}\ J\ s)(3 \times 10^8\ m/s) / (8 \times 10^{-14}\ J) = 2.5 \times 10^{-12}\ m$

$= 2.5 \times 10^{-3}\ nm$ gamma ray

7. $L = 23.5\, L_{sun} = 23.5\,(3.90 \times 10^{26}\ W) = 9.165 \times 10^{27}\ J/s$

$E = 9.165 \times 10^{27}\ J = \Delta m\, c^2 = \Delta m\,(3 \times 10^8\ m/s)^2 = \Delta m\,(9 \times 10^{16}\ m^2/s^2)$

$\Delta m = 1.02 \times 10^{11}\ kg$ (mass converted into energy)

$\mathcal{M} = \Delta m / 0.0071 = 1.43 \times 10^{13}\ kg$ (H converted to He)

8. $\tau_B / \tau_{sun} = \mathcal{M}_{sun}^3 / \mathcal{M}_B^3 = 1 / (2.3)^3 = 1 / 12.2 = 0.082$

$\tau_B = 0.082\, \tau_{sun} = 0.082 \times 10^{10}\ yr = 820 \times 10^6\ yr$

9. $\tau_B / \tau_A = \mathcal{M}_A^3 / \mathcal{M}_B^3 = (5)^3 / 1 = 125$ $\tau_B = 125\, \tau_A$

10a. $\tau = 1 / \mathcal{M}^3 = 1 / (9)^3 = 1.4 \times 10^{-3}$ **solar or**

$(1.4 \times 10^{-3}) (1.0 \times 10^{10}$ **years)** $=$ **14 million years**

10b. $\tau = 1 / \mathcal{M}^3 = 1 / (0.25)^3 = 64$ **solar or**

$(64) (1.0 \times 10^{10}$ **years)** $=$ **640 billion years**