

SOLUTION SET 9

Physics 2022

1. $m = +14.3$ $L^* / L_{\text{sun}} = 88$ $M_{\text{sun}} = +5.0$

$$M_{\text{sun}} - M^* = 2.5 \log (L^* / L_{\text{sun}}) = 2.5 \log (88) = 4.86$$

$$M^* = 5.0 - 4.86 = 0.14 \text{ mag}$$

$$m - M = 5 \log (d / 10) = 14.3 - 0.14 = 14.16$$

$$D = [\text{dex} (14.16 / 5)] (10) \approx 6800 \text{ pc}$$

2. $E = h c / \lambda = (6.63 \times 10^{-34} \text{ Js}) (3 \times 10^8 \text{ m/s}) / (0.21 \text{ m}) = 9.5 \times 10^{-25} \text{ J}$

$$(= 5.9 \times 10^{-6} \text{ eV})$$

3a. $P = 2\pi d / v = 2\pi (20,000 \text{ pc}) (3.086 \times 10^{13} \text{ km}) / [(400 \text{ km/s}) (60 \times 60 \times 24 \times 365.25)]$

$$P = 9.72 \times 10^{15} \text{ s} = 3.08 \times 10^8 \text{ year} = 308 \text{ million years}$$

3b. $\mathcal{M} = d^3 / P^2 = [(20,000 \text{ pc}) (206265 \text{ AU/pc})]^3 / [3.08 \times 10^8 \text{ yr}]^2$

$$\mathcal{M} = 7.4 \times 10^{11} \text{ solar masses}$$

4a. $P = 2\pi d / v = 2\pi (16,000 \text{ pc}) (3.086 \times 10^{13} \text{ km}) / [(270 \text{ km/s}) (60 \times 60 \times 24 \times 365.25)]$

$$P = 3.64 \times 10^8 \text{ yr}$$

4b. $\mathcal{M} = d^3 / P^2 = [(16,000 \text{ pc}) (206265 \text{ AU/pc})]^3 / (3.64 \times 10^8 \text{ yr})^2$

$$\mathcal{M} = 2.7 \times 10^{11} \text{ solar masses}$$

5a. $R_{\text{sch}} = 2 G M / c^2$

$$R_{\text{sch}} = 2(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2) (3.7 \times 10^6) (1.989 \times 10^{30} \text{ kg}) / (3 \times 10^8 \text{ m/s})^2$$

$$R_{\text{sch}} = 10.9 \times 10^9 \text{ m} = 10.9 \times 10^6 \text{ km} = 0.073 \text{ AU}$$

5b. $\sin \theta = D / d = (0.073 \text{ AU}) / [(8,000 \text{ pc}) (206265 \text{ AU/pc})] = 4.42 \times 10^{-11}$

$$\theta = 2.53 \times 10^{-9} \text{ degree} = 9.12 \times 10^{-6} \text{ arcsec} \quad \text{half-angle}$$

$$\text{full-angle} = 2 \theta = 1.8 \times 10^{-5} \text{ arcsec}$$

5c. $\sin \theta = D / d = (0.073 \text{ AU}) / [120 \text{ AU}] = 6.08 \times 10^{-4}$

$$\theta = 3.49 \times 10^{-2} \text{ degree} = 125 \text{ arcsec} \quad \text{half-angle}$$

$$\text{full-angle} = 2 \theta = 250 \text{ arcsec} \quad \text{Can be seen by naked eye}$$

6a. $C = 2 \pi r = 2 \pi (530 \text{ AU}) (1.5 \times 10^8 \text{ km/AU}) = 5.0 \times 10^{11} \text{ km}$

$$P = C / v = (5.0 \times 10^{11}) / 2500 \text{ km/s} = 2.0 \times 10^8 \text{ seconds} = 6.3 \text{ years}$$

6b. $(M_1 + M_2) = a^3 / P^2 = (530 \text{ AU})^3 / (6.3 \text{ yr})^2 = 3.8 \times 10^6 \text{ solar masses}$

7. $d = v / H_0 = (16,000 \text{ km/s}) / (70 \text{ km/s/Mpc})$

$$d = 229 \text{ Mpc} = 229 \times 10^6 \text{ pc} (\times 3.26 \text{ ly/pc}) = 745 \times 10^6 \text{ ly}$$

8. $d = v / H_0 = (10,800 \text{ km/s}) / (71 \text{ km/s/Mpc})$

$$d = 152 \text{ Mpc} = 152 \times 10^6 \text{ pc} = 496 \times 10^6 \text{ LY}$$

9. $m - M = 5 \log (d / 10) = +10 - (-19.9) = 29.9$

$$d = 9.5 \times 10^6 \text{ pc} = 9.5 \text{ Mpc}$$

Some possibly useful constants

$$1 \text{ pc} = 206,265 \text{ AU} = 3.26 \text{ ly}$$

$$1 \text{ AU} = 1.5 \times 10^8 \text{ km}$$

$$1 \text{ yr} = 3 \times 10^7 \text{ s}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$\sigma = 5.669 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

$$G = 6.7 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$\mathcal{M}_{\text{sun}} = 2.0 \times 10^{30} \text{ kg}$$

$$R_{\text{sun}} = 6.96 \times 10^5 \text{ km}$$

$$H_0 = 71 \text{ km/s/Mpc}$$