SOLUTION SET 10
Physics 2022

1. $m - M = 5 \log (d / 10) = +10 - (-19.9) = 29.9$

$d = 9.5 \times 10^6$ pc $= 9.5$ Mpc

2a. $d = v / H_o = (10{,}800$ km/s$) / (71$ km/s/Mpc$)$

$d = 152$ Mpc $= 152 \times 10^6$ pc $= 496 \times 10^6$ LY

2b. if $H_o < 70$ km/s/Mpc then d would increase

if $H_o > 70$ km/s/Mpc then d would decrease

3. $H_o = v / d = (7500$ km/s$) / (1.4 \times 10^2$ Mpc$) = 54$ km/s/Mpc

4. $(\mathcal{M}_1 + \mathcal{M}_2) = a^3 / P^2$

$= [\,(600 \times 10^3$ pc$)\,(206265$ AU/pc$)\,]^3 / (40 \times 10^9$ yr$)^2$

$= (1.89 \times 10^{33}) / (1.6 \times 10^{21}) = 1.2 \times 10^{12}$ solar masses

5. $H_o = 80$ km/s per Mpc $\quad v = 3600$ km/s

$v = d H_o$

$d = v / H_o = 3600$ km/s $/ 80$ km/s/Mpc $= 45.0$ Mpc

6. $d = 2$ pc $v = 100$ km/s

$P = 2\pi d / v = (2\pi \times 2 \times 206265 \times 1.5 \times 10^8 \text{ km}) / 100 \text{ km/s}$

$= 3.9 \times 10^{12}$ s $= 1.23 \times 10^5$ yr

$\mathcal{M} = d^3 / P^2 = (2\text{pc} \times 206265)^3 / (1.23 \times 10^5 \text{ yr})^2 = 4.6$ million solar masses

7. $T = 1 / H_o$

$H_o = 40$ km/s per Mpc

$$T = 1 / \{ [40 \text{ km/s/Mpc} / (10^6 \times 206265 \times 1.5 \times 10^8 \text{ km/Mpc})] (3600 \times 24 \times 365.25\text{s/yr}) \}$$

$= 1 / \{ (40 / 3.1 \times 10^{19}) (3.2 \times 10^7) \} = 1 / 4.1 \times 10^{-11}$ /yr

$T = 24.5 \times 10^9$ yr $(= 7.7 \times 10^{17}$ s)

8. $T = 20$ billion year

$$H_o = 1 / \{ (20 \times 10^9 \text{ yr})(3600 \times 24 \times 365.25\text{s/yr}) / [10^6 \times 206265 \times 1.5 \times 10^8 \text{ km/Mpc})] \}$$

$= 1 / \{ (20 \times 10^9) (3.2 \times 10^7) / (3.1 \times 10^{19}) \} = 1 / 2.04 \times 10^{-2}$ Mpc/s/km

$H_o = 49$ km/s/Mpc

9. $v / c = [(z + 1)^2 - 1] / [(z + 1)^2 + 1] = [(5.8 + 1)^2 - 1] / [(5.8 + 1)^2 + 1]$

$= [(6.8)^2 - 1] / [(6.8)^2 + 1] = [46.2 - 1] / [46.2 + 1] = 45.2 / 47.2$

$v = 0.958 c = 2.87 \times 10^5$ km/s

10. $v / c = [(z + 1)^2 - 1] / [(z + 1)^2 + 1] = [(8 + 1)^2 - 1] / [(8 + 1)^2 + 1]$

$= [(9)^2 - 1] / [(9)^2 + 1] = [81 - 1] / [81 + 1] = 80 / 82$

$v = 0.976\ c = 2.93 \times 10^5$ km/s

11. $R_{sch} = 2 G \mathcal{M} / c^2 = 2(6.67 \times 10^{-11}\ N\ m^2/kg^2)\ (10^9)\ (1.989 \times 10^{30}\ kg) / (3 \times 10^8\ m/s)^2$

$= 2.95 \times 10^{12}$ m $= 2.95 \times 10^9$ km $= 19.6$ AU

The average Sun-Pluto distance is 39.5 AU.